Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2223 – 7109

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of $[\mathrm{PQ}]$.

(a) Find the coordinates of M. [2]

Line L is perpendicular to $[\mathrm{PQ}]$ and passes through M.

(b) Find the gradient of L. [2]

(c) Hence, write down the equation of L. [1]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

2. [Maximum mark: 7]

The function f is defined by $f(x)=\dfrac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y=f(x)$, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

3. [Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on *The Dragon*	**Frequency**
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

(i) the probability that a randomly selected visitor will ride *The Dragon*;

(ii) the expected number of times a visitor will ride *The Dragon*. [4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand. [2]

4. [Maximum mark: 6]

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2\sin^2 x + \sin x - 1 = 0$. [1]

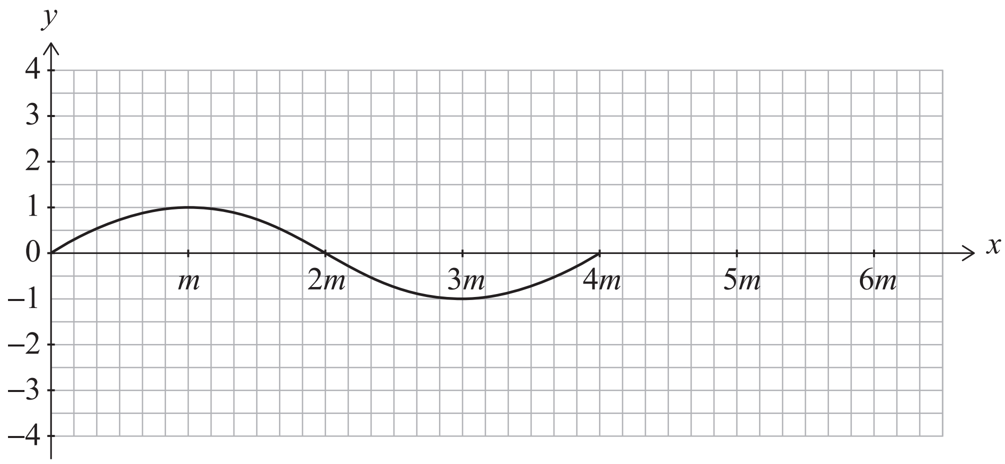
(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \le x \le \pi$. [5]

5. [Maximum mark: 6]

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^{x}$ has at least one real solution.

6. [Maximum mark: 6]

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0$, $2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

The function g is defined by $g(x) = 3\sin\dfrac{2qx}{3}$, for $0 \le x \le 6m$.

(b) On the axes above, sketch the graph of g. [4]

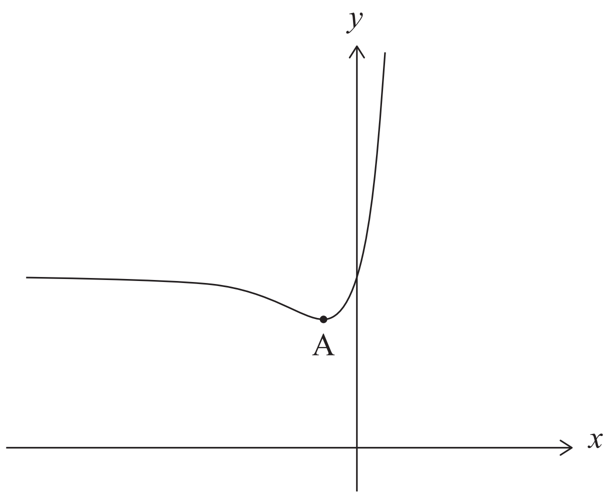
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The function h is defined by $h(x) = 2xe^{x} + 3$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of h, which has a local minimum at point A.



(a) Find the value of the y-intercept. [2]

(b) Find $h'(x)$. [2]

(c) Hence, find the coordinates of A. [5]

(d) (i) Show that $h''(x) = (2x + 4)e^{x}$.

(ii) Find the values of x for which the graph of h is concave-up. [4]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a) (i) Find the sum of the first five terms.

(ii) Given that $S_6 = 60$, find u_6. [4]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

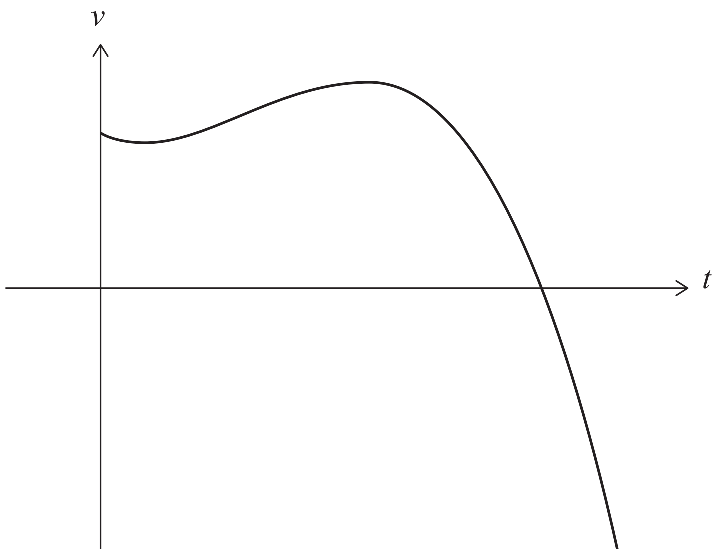
(e) Given that $v_{99} < 0$, find v_5. [2]

Do **not** write solutions on this page.

9. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \le t \le 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

(b) Find an expression for the acceleration of the object. [2]

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

(d) Find the greatest speed reached by the object for $0 \le t \le 4$. [2]

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

References:

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.